Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4427926590

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 Investigation and Modelling (Extended) **October/November 2021**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 3) and part **B** (Questions 4 to 6).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

DC (KS/SG) 212585/1

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 3)

GIRARD'S SUMS (30 marks)

You are advised to spend no more than 50 minutes on this part.

Albert Girard, a 17th century French mathematician, investigated numbers, N, that can be written as the sum of two squares, $a^2 + b^2$.
This task is about these numbers.

For this task, a and b are integers where $a \geqslant 0$ and $b \geqslant 0$.

1 **(a)** Complete the table.

a	a^2	b	b^2	$N = a^2 + b^2$	$N \div 4$	
2	4	6	36	40	10	remainder 0
18		10			106	remainder 0
28		16	256			remainder 0
4			64		20	remainder 0
	144		196		85	remainder 0
20	400			884	221	remainder 0
		0	0	900	225	remainder 0

[5]

(b) **(i)** When $a = 2$ and $b = 4$ then $N = 4k$, so N is a multiple of 4.

Find the value of k.

.... [2]

(ii) The values of a and b in the table are all even numbers.
When $a = 2m$ and $b = 2n$ then $N = 4k$.

Find an expression for k in terms of m and n.

.... [3]

(c) Not all multiples of 4 can be written as the sum of two square numbers.

Show that there are no values of a and b that give $k = 11$.

[2]

2 **(a)** Complete the table.

a	a^2	b	b^2	$N = a^2 + b^2$	$N \div 4$
7	49	5	25	74	18 remainder 2
21		19		802	200 remainder 2
17	289			914	remainder 2
			49	170	remainder 2
1		1			remainder

[4]

(b) When a is an odd number, $a = 2n - 1$.

(i) Use algebra to explain why, when a is an odd number, $a^2 \div 4$ has a remainder of 1.

.... [3]

(ii) Explain why, for the values in the table in **part (a)**, N is always $4k + 2$.

....

.... [2]

(c) When a and b are both odd, $N = 4k + 2$, so N is a multiple of 4 plus 2.
Not all multiples of 4 plus 2 can be written as the sum of two square numbers.

Find all the values of k from 1 to 9 where $N = a^2 + b^2$.

.... [3]

3 The values of N that can be written as the sum of two square numbers are of the form $4k+r$, where the remainder r is a constant.

(a) Explain why r can be 0, 1 or 2 but cannot be 3.

[3]

(b) $N = a^2 + b^2$

Find all the values of N, where $10 < N < 30$, that are of the form $4k+1$.

.... [3]

THE MODELLING TASK STARTS ON PAGE 8.

B MODELLING (QUESTIONS 4 to 6)

PRODUCTION BOUNDARIES (30 marks)

You are advised to spend no more than 50 minutes on this part.

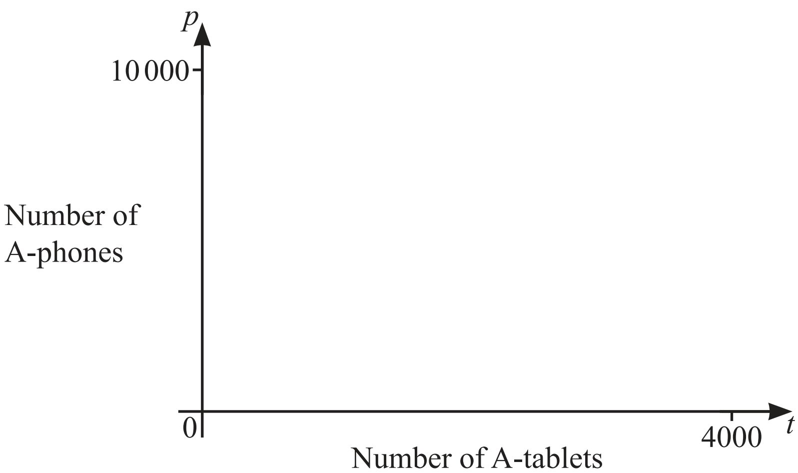
This task is about the number of computer tablets and mobile phones a company makes and sells.

The company owns two factories, A and B.
Factory A makes A-tablets and A-phones.
Factory B makes B-tablets and B-phones.

A *production boundary* is a curve or line.
Points on the curve or line are the maximum numbers of the two items a factory can make when all resources are used.
It is the boundary of the region which shows all the combinations of the two items a factory can make.

4 Factory A makes t A-tablets and p A-phones each day.
The manager of factory A uses the model $p = 9000 - \frac{t^2}{1000}$ where $t \geqslant 0$, as the production boundary for the output of A-tablets and A-phones.

(a) On the axes below, sketch this model.



[2]

(b) When factory A makes 9000 A-phones it cannot make any A-tablets.

Write down the maximum number of A-tablets it can make when it does not make any A-phones.

.... [1]

(c) On Monday, factory A makes 1000 A-tablets.
On Tuesday, factory A makes 1500 A-tablets.

Find the decrease in the maximum number of A-phones it can make from Monday to Tuesday.

.... [3]

(d) **(i)** On Wednesday, factory A makes 5000 A-phones.

Use your graph from **part (a)** to explain why it is not possible for it to make 2500 A-tablets on Wednesday.

....

.... [1]

(ii) On the graph in **part (a)** shade the region that represents the numbers of A-phones and A-tablets that factory A can make. [1]

(e) The company sells all the A-phones and A-tablets that factory A makes each day.
The company makes \$160 profit for each A-tablet and \$100 profit for each A-phone it sells.
The greatest possible daily profit at factory A is \$964 000.

(i) Write down a linear equation for this profit in terms of p and t.
Give your answer in the form $p = mt + c$.

.... [2]

(ii) Find the number of A-tablets and A-phones that factory A should sell in order to make a profit of \$964 000.

$t =$ ....

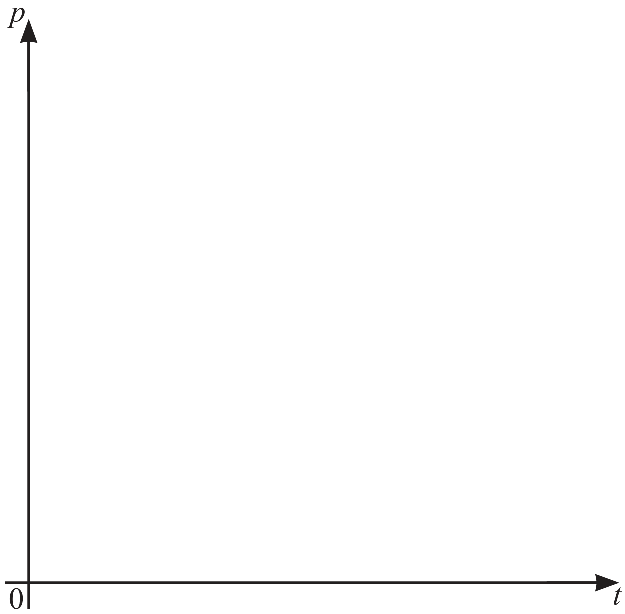
$p =$ .... [3]

5 Factory B makes t B-tablets and p B-phones.
The table shows the maximum numbers of B-phones that factory B can make each day for some numbers of B-tablets.

Number of B-tablets t	Number of B-phones p
1000	8000
2000	6000
3000	4000
4000	2000

As the number of B-tablets increases, the number of B-phones decreases at a constant rate.

(a) (i) Draw the production boundary for factory B on the axes below.



[2]

(ii) Find the equation which models this production boundary, giving p as a function of t.

.... [2]

(iii) Factory B makes at least 1000 B-tablets but no more than 4000 B-tablets each day.

Write down the domain of the model in **part (a)(ii)**.

.... [1]

(b) The company sells all the B-tablets and B-phones factory B makes each day.
The company makes \$200 profit for each B-tablet and \$190 profit for each B-phone it sells.
Each day, the manager of factory B expects to make the greatest possible profit.

(i) Find the greatest possible profit each day.

.... [3]

(ii) One day factory B has to make 2500 B-tablets.
On this day the profit is 73.3% of the greatest possible profit.

Work out the number of B-phones factory B makes on this day.

.... [4]

Question 6 is printed on the next page.

6 The company puts new machinery to make phones in factory A and factory B.

Factory A can now make double the number of A-phones.
Factory B can now make 10% more B-phones.
All other conditions remain the same.

(a) Complete the following models for the production boundaries at each factory after the changes.
Use the models in **Question 4** and **Question 5(a)**.

Factory A: $p =$ .... for $t \geqslant 0$

Factory B: $p = -2.2t + 11\,000$ for .... $\leqslant t \leqslant$ .... [2]

(b) After the changes, the greatest possible profit made each day by factory A is \$1 830 000.

Find the **total** greatest possible profit made each day by the company.

.... [3]